Formula Sheet for Physics 112

elementary charge $e = 1.6 \times 10^{-19} C$ mass of an electron $m_e = 9.1 \times 10^{-31} kg$, mass of a proton $m_p = 1.674 \cdot 10^{-27} kg$

charge of a proton: e, charge of an electron:$-e$

permittivity of the vacuum $\varepsilon_0 = 8.85 \cdot 10^{-12} \frac{C^2}{Nm^2} = 8.85 \cdot 10^{-12} \frac{F}{m}$

electron Volt: $1\ eV = 1.60 \times 10^{-19} J$

permeability of the vacuum: $\mu_0 = 4\pi \cdot 10^{-7} \frac{Tm}{A}$

Electrostatics:

electric charge: symbol q or Q, unit of charge: $C = coulomb$

electric force: between two point charges q_1, q_2 a distance r apart
$|\vec{F}| = k\dfrac{|q_1||q_2|}{r^2}$, $k = 8.99 \cdot 10^9 \dfrac{Nm^2}{C^2}$, unit of force: $N = newton$
like charges repel, unlike charges attract
electric field: $\vec{E} = \dfrac{\vec{F}}{q_0}$, unit of electric field: $\dfrac{N}{C}$
direction defined through the force on a *positive* test charge q_0
electric field generated by a point charge Q a distance r away: $|\vec{E}| = k\dfrac{|Q|}{r^2}$
force on a charge q in an electric field $\vec{E}$: $\vec{F} = q\vec{E}$

work done by the electric force when a charge moves from point A to point B:

$W_{AB} = EPE_A - EPE_B$, with the electric potential energy EPE

If only the electric force acts on a charge, its total energy $E_{total} = KE + EPE$ is conserved.
electric potential at a point: $V = \dfrac{EPE}{q_0}$, unit $volt = V$
$V_B - V_A = \dfrac{EPE_B}{q_0} - \dfrac{EPE_A}{q_0} = -\dfrac{W_{AB}}{q_0}$ is the electric potential difference between two points A and B
electric potential due to a point charge q at a point a distance r away: $V = k\dfrac{q}{r}$
for **uniform** electric fields: $V = Ed$

capacitors: capacitance: $C = \dfrac{q}{V}$, unit $F = farad, F = \dfrac{C}{V}$
energy stored: $= \dfrac{1}{2}qV = \dfrac{1}{2}CV^2 = \dfrac{1}{2}\dfrac{q^2}{C}$, energy density: $= \dfrac{1}{2}\varepsilon_0 E^2$

parallel plate capacitor: $C = \varepsilon_0 \frac{A}{d}$

if a dielectric material is inserted, the capacitance is multiplied by the dielectric constant κ: $C = \varepsilon_0 \kappa \frac{A}{d}$

Circuits:

current: $I = \frac{q}{t}$, unit: $A = ampere = \frac{C}{s}$

resistance: $R = \frac{V}{I}$, unit $\Omega = ohm = \frac{V}{A}$

resistance of a wire: $R = \rho \frac{L}{A}$

temperature dependence of resistance: $R = R_0[1 + \alpha(T - T_0)]$

power: $P = V \cdot I = I^2 R = \frac{V^2}{R}$, unit $W = watt = \frac{J}{s} = VA$, power $= \frac{\text{transferred energy}}{\text{time}}$

resistors in series: $R_{eq} = R_1 + R_2 + R_3 + ...$ resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$

for two resistors R_1, R_2 in series: $I_1 = I_2 = I$, $V_1 + V_2 = V$

for two resistors R_1, R_2 in parallel: $V_1 = V_2 = V$, $I = I_1 + I_2$

terminal voltage: $V_{ab} = \mathcal{E} - Ir$

Magnetism

magnetic field $\vec{B}$, unit: $1T = tesla = \frac{N}{Am}$

right hand rule (RHR-1) for force on positive charge: thumb points in direction of charge's velocity, index finger points in B field direction, middle finger points in force direction. Flip middle finger for force on negative charge.

right hand rule (RHR-1) for force on current carrying wire: thumb points in current direction, index finger points in B field direction, middle finger points in force direction.

force on charge in magnetic field: $F = qvB \sin\theta$

circular motion of charged particles in magnetic field: $r = \frac{mv}{qB}$

force on current carrying wire of length L in magnetic field: $F = ILB \sin\theta$

right hand rule (RHR-2) for direction of magnetic field due to a long straight wire: point thumb in direction of conventional current, fingers curl around wire, give direction of $\vec{B}$

magnetic field due to a long, straight wire: $B = \frac{\mu_0}{2\pi}\frac{I}{r}$

magnetic field at the center of a circular loop of radius R and N windings: $B = N\frac{\mu_0 I}{2R}$

magnetic field inside a solenoid: $B = n\mu_0 I$, n: number of turns per unit length

force per unit length on wire due to another wire a distance L away: $\frac{F}{\ell} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{L}$

parallel currents attract, antiparallel current repel

circular motion of charged particles in magnetic field: $r = \frac{mv}{qB}$

induced emf: $\mathcal{E} = -N\frac{\Delta\Phi_B}{\Delta t}$, magnetic flux $\Phi_B = BA\cos\theta$, θ is the angle that the magnetic field vector makes with the normal of the area A

Geometric Optics

all angles are measured with respect to the normal

angle of reflection = angle of incidence

concave mirror: positive focal length, convex mirror: negative focal length

Ray Tracing Rules for Mirrors:

For **concave** mirrors:

Ray 1: This ray is initially parallel to the principal axis and passes through the focal point after reflection.

Ray 2: This ray initially passes through the focal point and is reflected parallel to the principal axis.

Ray 3: This ray travels along a line that passes through the center of the mirror and reflects back on itself.

For **convex** mirrors:

Ray 1: This ray is initially parallel to the principal axis and appears to originate from the focal point after reflection.

Ray 2: This ray initially heads toward the focal point and is reflected parallel to the principal axis.

Ray 3: This ray travels along a line towards the center of the mirror and reflects back on itself.

Mirror equation: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$

Magnification: $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$

Sign conventions: object or image in front of the mirror have positive distances d_o and d_i

image behind the mirror has a negative image distance d_i

inverted image has a negative image height h_i

speed of light in a medium with refraction index n: $v = \frac{c}{n}$

Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$, angle of incident ray θ_1, angle of refracted ray θ_2

total internal reflection: $\sin\theta_c = \frac{n_2}{n_1}$

Thin Lenses

Power of a lens: $P = \frac{1}{f}$, unit: *diopters* $= D$

Lens equation: $\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f}$

Magnification: $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$

Sign Conventions:

f: focal length is positive for converging lenses, negative for diverging lenses

d_o: object distance; positive sign if object is on the same side as the light source (always the case, unless you have *combinations* of lenses)

d_i: image distance; positive sign if image is on the opposite side of the lens from where the light is coming, negative sign otherwise

h_o: object height; always positive

h_i: image height; positive if the image is upright, and negative if the image is reverted with respect to the object

Ray Tracing (3 principal rays)

1) **Converging Lens:** Draw a ray parallel to the axis, it will be refracted by the lens so that it goes through the focal point on the other side of the lens. **Diverging Lens:** Draw a ray parallel to the axis, it will be refracted by the lens so that it seems to come from the focal point on the object's side of the lens.

2) **Converging Lens:** Draw a ray that passes through the focal point on the same side of the lens as the object, it will emerge parallel to the axis on the other side of the lens. **Diverging**

Lens: Draw a ray that aims at the focal point on the other side of the lens, it will emerge parallel to the axis on the other side of the lens.

3) Draw a ray through the center, it will continue straight through the lens.

Mechanics

Equations for constant acceleration a in one dimension:

$v_x = v_{0x} + a_x t \qquad x = v_{0x} t + \frac{1}{2} a_x t^2$

$v_x^2 = 2a_x x + v_{0x}^2 \qquad v_{x,avg} = \frac{1}{2}(v_{0x} + v_x) \qquad v_{0x}$: initial velocity, i.e. velocity at $t = 0$

acceleration due to gravity at Earth's surface: $g = 9.8 m/s^2$

Newton's Second Law: $\sum \vec{F} = m\vec{a}$

Kinetic Energy: $KE = \dfrac{1}{2} m v^2$

Work - Kinetic Energy Theorem: $W_{net} = KE_f - KE_0 = \Delta KE$

Work: $W = F \cos\theta s$

gravitational potential energy: $GPE = mgh$

mechanical energy: $E_{mec} = KE + PE$ with $PE = GPE + EPE$

$W_{nc} = (KE_f + PE_f) - (PE_o + KE_o)$

Power $P = \dfrac{W}{t}$

Impulse: $\vec{J} = \vec{F}_{avg} \Delta t$

Impulse - Momentum Theorem: $\sum \vec{F}_{avg} \Delta t = m\vec{v}_f - m\vec{v}_0 = \vec{p}_f - \vec{p}_0$

Linear momentum: $\vec{p} = m\vec{v}$

If the sum of external forces acting on a system is zero, the system is called an isolated system.

For an isolated system, the total linear momentum is conserved: $\vec{P}_f = \vec{P}_0$

uniform circular motion - centripetal acceleration: $a = \dfrac{v^2}{r}$

centripetal force: $F = ma = m\dfrac{v^2}{r}$

period of uniform circular motion: $T = \dfrac{2\pi r}{v}$ frequency: $f = \frac{1}{T}$, unit: $Hz = hertz = \frac{1}{s}$

Quadratic equation: $ax^2 + bx + c = 0, \;\; x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vector $\vec{a}$: $a_x = a\cos(\theta)$, $\quad a_y = a\sin(\theta)$, $\quad \tan(\theta) = \frac{a_y}{a_x}$

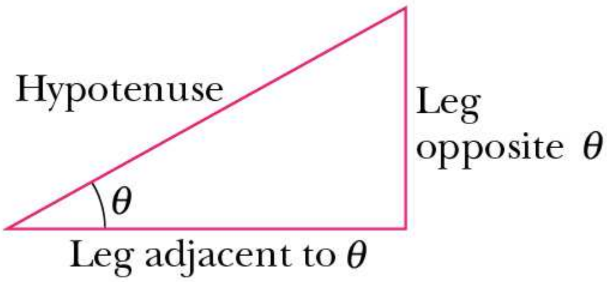
magnitude of a vector in the x-y plane: $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$

Trig and right triangle:

$$\sin\theta = \frac{\text{leg opposite } \theta}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{leg adjacent to } \theta}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{leg opposite } \theta}{\text{leg adjacent to } \theta}$$



Theorem of **Pythagoras**: $(\text{leg adjacent})^2 + (\text{leg opposite})^2 = (\text{hypotenuse})^2$

Unit vectors: $\hat{x}$ in x-direction $\quad \hat{y}$ in y-direction

Geometry

Area of a circle of radius r: $A = \pi r^2$

Circumference of a circle of radius r: $2\pi r$

Units: On this page of the formula sheet, you will find relations between units. The table contains only **SI - units**.

Note that once in a while, the same letter might either be the symbol for a physical quantity or the abbreviation for a unit, e.g. C may stand for capacitance, or for "Coulomb". This should always be clear from the context. If it is not clear to you, ask!

Non SI-unit: the electron-volt, eV, is a unit for energy often used for elementary particles. $1eV = 1.6 \cdot 10^{-19} J$.

physical quantity	symbol	unit	abbreviation for unit
displacement, distance	$\vec{x}, \vec{r}, ...$	meter	m
velocity, speed	$\vec{v}$, v	meter/second	$\frac{m}{s}$
acceleration	$\vec{a}$	meter/second/second	$\frac{m}{s^2}$
mass	m	kilogram	kg
force	$\vec{F}$	Newton	$N = \frac{kg\, m}{s^2}$
energy, work	E, PE, KE, W	Joule	$J = Nm$
momentum	$\vec{p}$	kilogram meter/second	$kg\frac{m}{s} = Ns$
impulse	$\vec{J}$	Newton second	Ns
electric charge	Q	Coulomb	C
electric field	$\vec{E}$	Newton/Coulomb	$\frac{N}{C} = \frac{V}{m}$
electric potential, voltage	V	Volt	V
capacitance	C	Farad	$F = \frac{C}{V}$
power	P	Watt	$W = \frac{J}{s} = VA = \frac{VC}{S}$
electric current	I	Ampere	$A = \frac{C}{s}$
resistance	R	Ohm	$\Omega = \frac{V}{A}$
resistivity	ρ	Ohm-meters	Ωm
magnetic field	$\vec{B}$	Tesla	$1T = \frac{N}{Am}$
power of a lens	P	diopter	$D = \frac{1}{m}$